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Instantiation for Theory Reasoning in Vampire

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Simplifing a Clause from Theory Axioms

1	initial	$14x \neq x^2 + 49 \vee p(x)$

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1	initial	$14x \neq x^2 + 49 \vee p(x)$
2	axiom	$x = (x - 1) + 1$

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Simplifing a Clause from Theory Axioms

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2	axiom	$x = (x - 1) + 1$
3	$1 + 2$	$14 \cdot x \neq ((x - 1) + 1) \cdot x + 49 \vee p(x)$

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3	$1 + 2$	$14 \cdot x \neq ((x - 1) + 1) \cdot x + 49 \vee p(x)$
4	axiom	$(x + 1) \cdot y = x \cdot y + y$

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3	$1 + 2$	$14 \cdot x \neq ((x - 1) + 1) \cdot x + 49 \vee p(x)$
4	axiom	$(x + 1) \cdot y = x \cdot y + y$
5	$3 + 4$	$14 \cdot x \neq ((x - 1) * x + x) + 49 \vee p(x)$

$\ldots$

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Simplifing a Clause from Theory Axioms

1	initial	$14x \neq x^2 + 49 \vee p(x)$
2	axiom	$x = (x - 1) + 1$
3	$1 + 2$	$14 \cdot x \neq ((x - 1) + 1) \cdot x + 49 \vee p(x)$
4	axiom	$(x + 1) \cdot y = x \cdot y + y$
5	$3 + 4$	$14 \cdot x \neq ((x - 1) * x + x) + 49 \vee p(x)$
		$\ldots$
(much later)		$p(7)$

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Alternative: Instantiation

Suppose we guess $x = 7$:

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Alternative: Instantiation

Suppose we guess $x = 7$:

$$14 \cdot 7 \neq 7^2 + 49 \vee p(7)$$

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Alternative: Instantiation

Suppose we guess $x = 7$:

$$14 \cdot 7 \neq 7^2 + 49 \vee p(7)$$

$$\downarrow\!\!\!\!\sim \qquad \text{evaluate}$$

$$98 \neq 98 \vee p(7)$$

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Alternative: Instantiation

Suppose we guess $x = 7$:

$$14 \cdot 7 \neq 7^2 + 49 \vee p(7)$$

$\downarrow\!\!\!\downarrow$ evaluate

$$98 \neq 98 \vee p(7)$$

$\downarrow\!\!\!\downarrow$ remove trivial inequality

$$p(7)$$

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Alternative: Instantiation

- Find instance that makes theory part of a clause false
- Substitute and delete theory part
- Rule

$$\frac{P \vee D}{D\theta}$$ theory instance

  - P pure (all constant symbols have a fixed interpretation)
  - $P\theta$ unsatisfiable in the theory

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Alternative: Instantiation

- Why pure?

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Alternative: Instantiation

- Why pure?

  $\Rightarrow$ We pass $\neg P$ to an SMT solver!

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Alternative: Instantiation

- Why pure?

  $\Rightarrow$ We pass $\neg P$ to an SMT solver!
- $\neg P$ has a model: construct θ from model
  - $14x = x^2 + 49$ has a model for $x = 7$
  - $\theta = \{x \mapsto 7\}$

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Alternative: Instantiation

- Why pure?

  $\Rightarrow$ We pass $\neg P$ to an SMT solver!
- $\neg P$ has a model: construct θ from model
  - $14x = x^2 + 49$ has a model for $x = 7$
  - $\theta = \{x \mapsto 7\}$
- Model construction needs purity (for now)

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Abstraction

- Suppose we want to resolve

$$r(14y)$$

$$\neg r(x^2 + 49) \vee p(x)$$

$\Rightarrow$ No pure literals

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Abstraction

- Suppose we want to resolve

$$r(14y)$$

$$\neg r(x^2 + 49) \vee p(x)$$

$\Rightarrow$ No pure literals

- Abstract to

$$z \neq 14y \vee r(z)$$

$$u \neq x^2 + 49 \vee \neg r(u) \vee p(x)$$

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Problems with Abstraction

- Eager application too expensive, fold into unification

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Problems with Abstraction

- Eager application too expensive, fold into unification
- Instantiation undoes abstraction:

$$p(1,5)$$

$\rightsquigarrow$ abstract

$$x \neq 1 \vee y \neq 5 \vee p(x,y)$$

$\rightsquigarrow$ instantiate

$$p(1,5)$$

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Trivial Literals

- Form: $x \neq t$ (x not in t)
- Pure
- x only occurs in other trivial literals or other non-pure literals

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Updated Rule

$$\frac{P \vee D}{D\theta} \text{ theory instance}$$

- $P\theta$ unsatisfiable in the theory
- P pure
- P does not contain trivial literals

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Improvements to Vampire

SMT-LIB

Logic	*New solutions*	*Uniquely solved*
ALIA	*1*	*0*
LIA	*14*	*0*
LRA	*4*	*0*
UFDTLIA	*5*	*0*
UFLIA	*28*	*14*
UFNIA	*13*	*4*

TPTP

Category	*New solutions*	*Uniquely solved*
ARI	*13*	*0*
NUM	*1*	*1*
SWW	*3*	*1*

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Future Work

- What about multiple solutions?
- What about inequalities ($9 < x^2$ vs. $9 \neq x^2$)
- What about uninterpreted symbols?

Vampire at `https://github.com/vprover/vampire`

Thanks!